General Relativity

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1 General Relativity

1.1 Why Force is Like Curvature

- Alice and Bob on the earth's equator, at a large yelling distance away from each other
- Earth is frictionless
- Pushed straight north
- Eventually will impact each other as the great circles they move along collide with each other
- Possibilities

  Force Bob thinks his path is being curved by some outside force

$$F_0 = -m\left(\frac{v}{B}\right) l(t) \tag{1}$$

$$B = 6400KM \tag{2}$$

  Curvature Alice thinks they are travelling along a curved surface

$$\text{radius of curvature} = R_c = B \tag{3}$$

$$\theta = \frac{l_0}{r_0} = \frac{l_o}{R_E} = \frac{l(t)}{r(t)} \tag{4}$$

$$F = ma = m\frac{d^2}{dt^2}l(t) \tag{5}$$

$$r(t) = R_E \cos(\phi(t)) \tag{6}$$

$$l(t) = \frac{l_0}{R_E} r(t) \tag{7}$$

$$= \frac{l_0}{R_E} R_E \cos(\phi(t)) \qquad \text{substitute in } r(t) \tag{8}$$

$$= l_0 \cos\left(\frac{vt}{R_E}\right) \qquad \text{Cancel and substitute in } \phi(t) \tag{9}$$

$$\phi(t) = S/R_E = vt/R_E \tag{10}$$

$$F = -m\left(\frac{v}{R_E}\right)^2 l(t) \tag{11}$$

1.2 Equivalence Principle

- It is always possible, for some object and force, to replace a force by some equivalent curvature
- For gravity, this works particularly well

$$m_i = \text{inertial mass} \tag{12}$$
$$m_g = \text{gravitational mass} \tag{13}$$
$$F = m_i a \tag{14}$$
$$F_N = -\left(\frac{GM}{r^2}\right) m_g \tag{15}$$
$$m_i a = \frac{GM}{r^2} m_g \tag{16}$$
$$a = \left(\frac{GM}{r^2}\right) \frac{m_g}{m_i} \tag{17}$$
$$m_g = m_i \tag{18}$$
$$\tag{19}$$

All bodies fall with equal acceleration

- GR = SR + Newton's Law + Straight Lines

2 Topology and General Relativity

2.1 Topology

- Intuition: Vectors — more than just numbers, has other invariants even if you try to change the coordinates by rotating (eg, length)
- Imagine a surface like a sphere

$$R^2 = x^2 + y^2 + z^2 \tag{20}$$
$$x' = zx \tag{21}$$
$$y' = y/2 \tag{22}$$
$$z' = 3z \tag{23}$$
$$R^2 = \left(\frac{x'}{2}\right)^2 + (2y')^2 + \left(\frac{z'}{3}\right)^2 \tag{24}$$

Distorting the coordinates give you an ellipsoid

- Invariants

  Handles donut, coffee cup, torus all have one (basically a big hole that you could grab on to with

  Holes If you remove one point from a sphere, you get something equivalent to a plane

  Twists Möbius strip has one

2.2 Requirements for GR

- All observers are equal (as long as gravity is the only force in effect)
- Alternatively, all coordinate systems are equivalent
- Hence, need things that are invariant with respect to coordinate system
  - Topology

  - Events

- What is the difference between a donut and a coffee cup? Geometry
  - Straight lines (or just distances) (metric = g)
  - Derivatives (connection = Γ)
- In GR, $g \leftrightarrow \Gamma$, but not in quantum gravity
- What you need for GR:
  - Topology
  - Matter
  - Coordinate system

  Plug into Einstein equations, get a a metric g

3 Black Holes

3.1 Schwarzschild Black Hole

- Stuff for GR

  Topology $\mathbb{R}^4$ – line (three space minus the point for the the singularity, crossed with a line for time: $(\mathbb{R}^3 - \{\text{point}\}) \oplus \mathbb{R}^1$)

  Matter Spherically symmetric

  Coordinate system Far away from matter (fiducial observer = "Fido")

- Get metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(\frac{1}{1 - \frac{2M}{r}}\right) dr^2 + \cdots \tag{25}$$

- Meaning
  - ds = space-time distance
  - dr = physical distance
  - dt = time distance
- Problems
  - $r = 0$: always an issue
  - $r = 2M$: only because of coordinates

3.2 Sonic Black Hole

- Take a huge pond
- Drill a hole in the bottom, with a spike in the bottom
- Water flows out of the hole faster than the speed of sound in the water
- Take tadpoles in the pool

  Freefo Free fall observer

  Fido Fiduciary observer

- Able to "see" solely using sonar
- Equipped with a regular noisemaker (eg, loud heartbeat)

- As Freefo falls in, Fido hears Freefo's heart slowing down
- Past the horizon where the water starts moving faster than the speed of light, Freefo's heart beat (which travels as waves in the water) will never get out
- From Freefo's point of view, the horizon isn't anything too special

4 Semi-classical GR

- Hawking radiation
  - $\Delta x \Delta p = \hbar$
  - Near the black hole, uncertainty about where exactly things are
  - Because particles spontaneously appear and disappear, some energy will escape from the black hole
  - Eventually the black hole will completely evaporate (maybe, if insufficient matter gets sucked in)
  - Black holes have a "temperature": $T_{BH} = \frac{\hbar c^3}{\delta \pi G M k_B}$ is proportional to $\frac{1}{M}$
  - Information entropy
    - Disorder
    - Classical: Black hole is perfectly ordered, entropy = 0
    - Quantum: Black hole has screwy boundary:

$$\text{entropy} = S = \text{Information (bits)} = \frac{1}{4}\left(\frac{A}{l_{pl}^2}\right) \tag{26}$$

$$l_{pl} = \text{scale where spacetime loses its meaning} \tag{27}$$

$$\Delta x \Delta p = \hbar \tag{28}$$

$$\Delta p = p \tag{29}$$

$$\Delta x = l \tag{30}$$

$$\therefore pl = \hbar \tag{31}$$

$$\therefore p = \hbar / l \tag{32}$$

$$p = \frac{E}{c} = \frac{m_g c^2}{c} = m_g c \tag{33}$$

$$m_g = p/c = \frac{\hbar}{ec} \tag{34}$$

$$l = \frac{R_{swh}}{2} = \frac{G m_g}{c^2} = \frac{G\hbar}{l c^3} \tag{35}$$

Multiplying by l and taking the root,

$$l = \sqrt{\frac{G\hbar}{c^3}} = l_{pl} \tag{36}$$

$$\tag{37}$$

So, GR and quantum act strangely together, because we can randomly get black holes

  - Now we get things quantized at the size of l_{pl}, so we have just one bit per small region, instead of one bit per point

5 Quantum Gravity and Holography

5.1 Holography

- All these black holes reduce our freedom
- Final entropy:

$$S_f = \frac{1}{4}\frac{A}{l_{pl}^2} \tag{38}$$

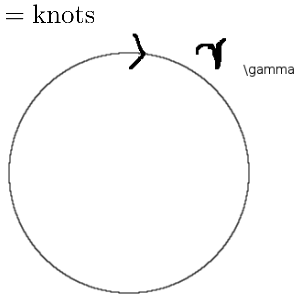
- By the second law of thermodynamics,

$$S_i \leq S_f = \frac{1}{4}\frac{A}{l_{pl}^2} \tag{39}$$

- So, the information is proportional to the *surface area*, so is the world 2D?

5.2 Quantum Gravity

- Look at (2D space + 1D time = 3D spacetime)
- This has been solved
- Observables = integrals over path in spacetime
  $H[\gamma]$ = knots



- Quantum gravity in 2 space dimensions is holographic

5.3 String Theory

- Lives on a surface in 10 dimensions (4 space dimensions, 1 time dimension, plus a 5 spere at every point in space)
- Drop a dimension, because we look at the boundary (3+1 dim)
- Correspondence between String theory and Conformal Field Theory