Blackholes

Michael (ISSYP lecturer), filtered through Alex Dehnert

Thursday, August 23, 2007

Recommended book: *Gravity From the Ground Up* by Schutz (ISBN: 0 521 45506 5). How gravity works, starting with Newton and working forward

Book from the lecture: *Gravitation* by Meisner (sp?) (available online)

1 Laws of Gravity

1.1 Newton

$$m_i \frac{d^2\vec{x}}{dt^2} = \vec{F} = m_g\vec{g} \tag{1}$$

$$\vec{g} = \sum_j \frac{-Gm_i(\vec{x} - \vec{x_j})}{|\vec{x} - \vec{x_j}|} \tag{2}$$

$$= \nabla\phi(\vec{x_j} \tag{3}$$

$$\phi(\vec{x_j}) = -G \int \frac{\rho(\vec{x_j})d^3x}{|\vec{x_j} - x|} \tag{4}$$

$$\nabla^2\phi = 4\pi G\rho(\vec{x_j}) \tag{5}$$

1.2 Einstein

$$r = \frac{GM}{3c^2} \tag{6}$$

$$\text{Let } M = \frac{4}{3}\pi r^2\rho \qquad \text{(where } \rho \text{ is uniform density (average over r))} \tag{7}$$

$$f_r = f_e(1 + \frac{gH}{c^2} \tag{8}$$

$$\tag{9}$$

2 Dark Stars

Rev John Michell (1783) suggests that a giant star could keep even light from escaping.

Escape velocity

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \tag{10}$$

$$v = \sqrt{\frac{2GM}{r}} \tag{11}$$

3 Einstein's Ideas

Equivalence principle: Can't distinguish a smoothly accelerating elevator (or rocket) from gravity pulling on you.

Gravity as curvature: Gravity curves light path and time (slows it down): $G_{uv} + \Lambda g_{uv} = 8\pi T_{uv}$ (G = geometry of space time, Λ = cosmological constant, T = distribution of matter, motion in spacetime)

3.1 Spacetime

Constant time = hyperbola

$$(c\,t_B)^2 = (c\,t_A)^2 + X_A^2 \tag{12}$$

$$\tag{13}$$

Mathematical tool that handles space and time $P(t, x, y, z)$.
Ex: 2D space:

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 \qquad \text{Euclidean Cartesian space} \tag{14}$$

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2 \qquad \text{Minkowski Cartesian space} \tag{15}$$

$$\tag{16}$$

A body will move along the curve of shortest distance (geodesic) in *spacetime* unless a force acts on it (not generally shortest distance in normal space).

In general, geodesics are difficult to calculate.

Easy if you embed in Euclidean space.

Metric: converts flat map distances into actual distances. Denoted by "g".

Finding a useful metric is hard — then you just slot it into a standard equation.

Non-physics example: Taxi metric to convert time, location, etc. into a cab ride cost.

Special relativity: Matter moves at most as fast as light. Stuck inside a cone limiting speed.

3.2 Metric g in four dimensions

In four dimensions, need 10 numbers to indicate curvature.
Need additional math tools to keep them straight.
Call it a *tensor*. Organize groups of tensors.

- 0-tensor: single number, eg 5
- 1 tensor: 4 numbers (vector), eg $A_j = (1, 0, -\pi, 2)$
- 2-tensor: 4x4=16 number (matrix), eg... eh, you know...

$$G_{uv} + \Lambda g_{uv} = 8\pi T_{uv} \tag{17}$$

$$G_{uv} = R_{ij} - \frac{1}{2} g_{uv} R \qquad \text{(Einstein's)} \tag{18}$$

$$T_{uv} = \text{stress-energy tensor — describes the density, flux of energy and momentum in spacetime} \tag{19}$$

$$g_{uv} = \text{metric tensor} \tag{20}$$

EFE is a tensor equation relating some 4x4 tensors. Einstein's equations are actually 16 equations of form: $G_{11} = 8\pi G T_{11} + \Lambda g_{11}$

3.3 Verification of GR

In 1919, the sun was seen to have distorted the light from a distant star by about 1.64 arc-seconds. Needed to be done during a solar eclipse because otherwise there would be too much light.